

Implications of TeV scale $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ quark-lepton unification

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(Received 17 March 1999; published 28 October 1999)

The alternative $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ gauge model, which allows unification of the quarks and leptons at the TeV scale, is studied in detail. We discuss the implications for nucleon decay, B and K rare meson decays, and neutrino masses. We also explain how this model solves the gauge hierarchy problem without using supersymmetry or extra large dimensions. [S0556-2821(99)07519-0]

PACS number(s): 12.60.Fr, 13.20.-v, 13.30.-a, 14.60.Pq

I. INTRODUCTION

There has been some discussion lately about the gauge hierarchy problem [1]. We would like to contribute to this discussion by suggesting the following rather simple solution: If the scale of new physics is M_{new} then a gauge hierarchy can be avoided provided that M_{new} is not too different from the weak scale, M_{weak} . This suggests the following rough upper limit:

$$M_{\text{new}} \lesssim \text{few TeV}. \quad (1)$$

We prefer to exclude gravity from our discussion for the obvious reason that it is not a well understood quantum theory. Despite the large value of $M_P \sim 10^{19}$ GeV it is not clear whether this poses a fine-tuning problem or not. The mere existence of the two disparate scales M_{weak} and M_P does not *necessarily* imply a fine-tuning problem, just like the existence of the disparate scales Λ_{QCD} and M_{weak} does not imply a fine-tuning problem in the standard model. Thus, we argue that so long as $M_{\text{new}} \lesssim \text{few TeV}$ the gauge hierarchy problem can be avoided. Of course, it should also be emphasised that the condition, Eq. (1), is of great practical importance since it means that the theory can be subject to many experimental tests (in principle).

Given the rather stringent requirement [Eq. (1)] one might imagine that there is no new physics beyond the standard model. We argue that this is unlikely for at least three reasons.

(1) There is experimental and theoretical evidence for neutrino masses. The experimental evidence comes from the neutrino physics anomalies [such as the atmospheric, solar, Liquid Scintillation Neutrino Detector (LSND) anomalies], while the theoretical evidence comes from the electric charge quantization problem of the minimal standard model [2].

(2) Each generation contains five distinct fermionic gauge multiplets.

(3) The standard model is a bit ugly because it contains 20 theoretically unconstrained parameters.

Let us first remark that the model to be discussed in this paper has many more parameters than the standard model, so that we have certainly not made any progress on the parameter problem. However, the model does partially address the

other two points identified above. One of the reasons that each generation contains five distinct fermion multiplets is that the quarks and the leptons are similar but lack any real symmetry in the standard model. Thus one obvious way to improve on this is to embed the standard model into a gauge model with a symmetry between the quarks and the leptons. Given the constraint, Eq. (1), there are only two possibilities that we are aware of. The first TeV scale quark-lepton unified model was proposed in Ref. [3] where a leptonic $SU(3)_l$ color group was assumed so that a discrete Z_2 quark-lepton symmetry can be defined [the $SU(3)_l$ gauge symmetry is assumed to be spontaneously broken at the TeV scale]. More recently, one of us [4] has also shown that it is possible to modify the usual Pati-Salam model [5] such that the quarks and the leptons can be unified with gauge group $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ at the low scale of about a TeV.¹ The purpose of this paper is to provide a systematic study of this alternative $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ model (or 422 model for short).

The outline of this paper is as follows: In Sec. II we review the basic structure of the alternative $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ model. In Sec. III we investigate nucleon decay in this model. As already noted in Ref. [4], gauge interactions conserve a global baryon number, however this symmetry can be broken by scalar interactions in the Higgs potential. We show that the effect of the scalar mediated nucleon decay is to induce neutron decay $N \rightarrow l^+ l^- \nu$ (where $l = e, \mu$). We provide a rough estimate of this decay rate which we show is consistent with a TeV symmetry breaking scale. In Sec. IV we discuss rare B , K meson decays. These decays provide the main experimental bound on the model. In Sec. V we discuss neutrino masses in the model which are naturally small, despite the TeV symmetry breaking scale. In Sec. VI we conclude.

II. THE ALTERNATIVE 422 MODEL

In this section, we review the alternative 422 model. For more details see Ref. [4]. The gauge symmetry of the model is

¹For some discussions of the usual Pati-Salam model with a low symmetry breaking scale, see Ref. [6]. Note however that in the usual Pati-Salam model the lowest possible value for the symmetry breaking scale is still quite high. According to Ref. [7] it is $m_{W'} \gtrsim 13$ TeV.

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$$SU(4) \otimes SU(2)_L \otimes SU(2)_R. \quad (2)$$

Under this gauge symmetry the fermions of each generation transform in the anomaly free representations:

$$Q_L \sim (4, 2, 1), \quad Q_R \sim (4, 1, 2), \quad f_L \sim (1, 2, 2). \quad (3)$$

The minimal choice of scalar multiplets which can both break the gauge symmetry correctly and give all of the charged fermions mass is

$$\chi_L \sim (4, 2, 1), \quad \chi_R \sim (4, 1, 2), \quad \phi \sim (1, 2, 2). \quad (4)$$

These scalars couple to the fermions as follows:²

$$\begin{aligned} \mathcal{L} = & \lambda_1 \bar{Q}_L (f_L)^c \tau_2 \chi_R + \lambda_2 \bar{Q}_R f_L \tau_2 \chi_L + \lambda_3 \bar{Q}_L \phi \tau_2 Q_R \\ & + \lambda_4 \bar{Q}_L \phi^c \tau_2 Q_R + \text{H.c.}, \end{aligned} \quad (5)$$

where the generation index has been suppressed and $\phi^c = \tau_2 \phi^* \tau_2$. Under the $SU(3)_c \otimes U(1)_T$ subgroup of $SU(4)$, the 4 representation has the branching rule, $4 = 3(1/3) + 1(-1)$. We will assume that the $T = -1$, $I_{3R} = 1/2$ ($I_{3L} = 1/2$) components of χ_R (χ_L) gain nonzero vacuum expectation values (VEVs) as well as the $I_{3L} = -I_{3R} = -1/2$ and $I_{3L} = -I_{3R} = 1/2$ components of the ϕ . We denote these VEVs by $w_{R,L}$, $u_{1,2}$, respectively. In other words,

$$\begin{aligned} \langle \chi_R(T = -1, I_{3R} = 1/2) \rangle &= w_R, \\ \langle \chi_L(T = -1, I_{3L} = 1/2) \rangle &= w_L, \\ \langle \phi(I_{3L} = -I_{3R} = -1/2) \rangle &= u_1, \end{aligned}$$

$$\langle \phi(I_{3L} = -I_{3R} = -1/2) \rangle = u_2. \quad (6)$$

We will assume that the VEVs satisfy $w_R \gg u_{1,2}$, w_L so that the symmetry is broken as follows:

$$\begin{aligned} & SU(4) \otimes SU(2)_L \otimes SU(2)_R \\ & \downarrow \langle \chi_R \rangle \\ & SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\ & \downarrow \langle \phi \rangle, \langle \chi_L \rangle \\ & SU(3)_c \otimes U(1)_Q \end{aligned} \quad (7)$$

where $Y = T + 2I_{3R}$ is the linear combination of T and I_{3R} which annihilates $\langle \chi_R \rangle$ (i.e., $Y \langle \chi_R \rangle = 0$).³ Observe that in the limit where $w_R \gg w_L$, u_1, u_2 , the model reduces to the standard model. The VEV w_R breaks the gauge symmetry to the standard model subgroup. This VEV also gives large $SU(2)_L \otimes U(1)_Y$ invariant masses to an $SU(2)_L$ doublet of exotic fermions, which have electric charges $-1, 0$. We will denote these exotic fermions with the notation E^-, E^0 . These exotic fermions must have masses greater than about $m_Z/2$ otherwise they would contribute to the Z width. Observe that the right-handed chiral components of the usual charged leptons are contained in Q_R . They are the $T = -1$, $I_{3R} = -1/2$ components. The usual left-handed leptons are contained in f_L along with the right-handed components (CP conjugated) of E^0, E^- . It is instructive to write out the fermion multiplets explicitly. For the first generation,

$$Q_L = \begin{pmatrix} d_y & u_y \\ d_g & u_g \\ d_b & u_b \\ E^- & E^0 \end{pmatrix}_L, \quad Q_R = \begin{pmatrix} d_y & u_y \\ d_g & u_g \\ d_b & u_b \\ e & \nu \end{pmatrix}_R, \quad f_L = \begin{pmatrix} \nu_L & (E_R^-)^c \\ e_L & (E_R^0)^c \end{pmatrix}_L, \quad (8)$$

and similarly for the second and third generations. In the above matrices the first column of $Q_L(f_L, Q_R)$ is the $I_{3L}(I_{3R}) = -1/2$ component while the second column is the $I_{3L}(I_{3R}) = 1/2$ component. The four rows of Q_L, Q_R are the four colors and the rows of f_L are the $I_{3L} = \pm 1/2$ components. Observe that the VEVs $w_L, u_{1,2}$ have the quantum numbers $I_{3L} = -1/2, Y = 1$ (or equivalently $I_{3L} = 1/2, Y$

$= -1$). This means that the standard model subgroup, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is broken to $SU(3)_c \otimes U(1)_Q$ in the usual way (with $Q = I_{3L} + Y/2 = I_{3L} + I_{3R} + T/2$).

III. SCALAR χ_R AND χ_L MEDIATED BARYON-NUMBER VIOLATING INTERACTIONS

One of the main constraints on unified models is the empirical limit on nucleon decay. Baryon charge in the alterna-

²Note that we do not include a bare mass term $m_{\text{bare}} \bar{f}_L (f_L)^c$, although such a term is allowed by the gauge symmetry of the model. We assume that $m_{\text{bare}} \ll M_{\text{weak}}$ so that it can be neglected. This is not unreasonable, since the bare mass scale is completely independent of the weak scale.

³In Ref. [4] a distinct but physically equivalent convention was used, which leads to $Y = T - 2I_{3R}$. The difference is just a $SU(2)_R$ rotation.

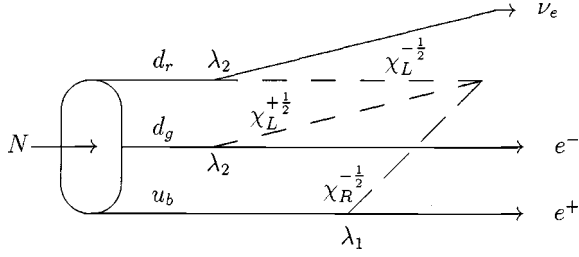


FIG. 1. Feynman diagram for the scalar mediated neutron decay $N \rightarrow e^+ e^- \nu_e$.

tive 422 model is defined as $B = (B' + T)/4$ where the B' charges of Q_L , Q_R , $\chi_{L,R}$ are all +1 and the B' charges of f_L , ϕ are 0. (The B' charges of the gauge bosons are also zero). This baryon charge is conserved by the gauge interactions and Yukawa Lagrangian, Eq. (5). [While B' and T are both broken by the vacuum, the combination $B' + T$ is unbroken, since $(B' + T)\langle\chi_R\rangle = (B' + T)\langle\chi_L\rangle = (B' + T)\langle\phi\rangle = 0$.]⁴ Thus, the only part of the Lagrangian which can potentially mediate nucleon decay is the Higgs boson potential. It is quite easy to see that the only gauge invariant and renormalizable terms in the Higgs boson potential which break the $U(1)_B$ symmetry are

$$V_1 = \tilde{\lambda}_1 \chi_L^{-1/2} \chi_L^{+1/2} \chi_R^{-1/2} \chi_R^{+1/2} + \tilde{\lambda}_2 \chi_L^{-1/2} \chi_L^{+1/2} \chi_L^{-1/2} \chi_L^{+1/2} + \tilde{\lambda}_3 \chi_R^{-1/2} \chi_R^{+1/2} \chi_R^{-1/2} \chi_R^{+1/2} + \text{H.c.}, \quad (9)$$

where the $I_{3L,R}$ quantum numbers have been explicitly shown as superscripts. From Eq. (5) the color triplet components of the χ 's interact with the fermions as follows:

$$\begin{aligned} \mathcal{L}^\chi = & \lambda_1 \chi_R^{-1/2} [\bar{d}_L(\nu_L)^c - \bar{u}_L(e_L)^c] + \lambda_1 \chi_R^{+1/2} (\bar{d}_L E_R^- - \bar{u}_L E_R^0) \\ & + \lambda_2 \chi_L^{-1/2} [\bar{d}_R \nu_L + \bar{u}_R (E_R^-)^c] \\ & - \lambda_2 \chi_L^{+1/2} [\bar{d}_R e_L + \bar{u}_R (E_R^0)^c] + \text{H.c.} \end{aligned} \quad (10)$$

Thus, the Higgs potential term which leads to the most significant nucleon decay is expected to be⁵

$$\begin{aligned} V_2 = & \tilde{\lambda}_1 \chi_L^{-1/2} \chi_L^{+1/2} \chi_R^{-1/2} \langle\chi_R^{+1/2}\rangle + \text{H.c.} \\ = & \tilde{\lambda}_1 w_R \epsilon^{ijk} \chi_{Li}^{-1/2} \chi_{Lj}^{+1/2} \chi_{Rk}^{-1/2} + \text{H.c.}, \end{aligned} \quad (11)$$

where we have made the $SU(3)_c$ indices explicit $[(i, j, k) \in 1, 2, 3]$. This term mediates neutron decay via the Feynman diagram in Fig. 1.

⁴Note that the baryon charge $[B = (B' + T)/4]$ of the quarks is 1/3 and the baryon charge of the leptons is zero. Also it is straightforward to check that the baryon charge of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge bosons are also zero.

⁵Observe that the term $\chi_L^{-1/2} \chi_L^{+1/2} \chi_L^{-1/2} \langle\chi_L^{+1/2}\rangle$ primarily mediates $\Delta^-(ddd)$ decay, while $\chi_R^{-1/2} \chi_R^{+1/2} \chi_L^{-1/2} \langle\chi_L^{+1/2}\rangle$, $\chi_R^{-1/2} \chi_R^{+1/2} \chi_R^{-1/2} \langle\chi_R^{+1/2}\rangle$ terms can mediate nucleon decay but are suppressed because the $\chi_R^{+1/2}$ state couples to the weak-eigenstate E (which contains only a tiny admixture of the light e, μ mass eigenstates).

The matrix element will contain a term for the propagator of each scalar χ which will contribute a factor of m_χ^{-2} . Thus the matrix element will be proportional to $m_{\chi_R}^{-2} m_{\chi_L}^{-4}$ and the decay time, being proportional to the inverse square of the matrix element will be of the form

$$\tau_N \sim \left(\frac{4\pi}{\tilde{\lambda}_1 w_R} \right)^2 \lambda_1^{-2} \lambda_2^{-4} m_{\chi_R}^4 m_{\chi_L}^8 m_N^{-11}, \quad (12)$$

where λ_1 and λ_2 are the dimensionless coupling constants from the interaction Lagrangian equation (5), $\tilde{\lambda}_1 w_R$ is from the trilinear scalar interaction term, Eq. (11). The neutron mass m_N has been introduced as a dimensional factor because we are studying neutron decay. Observe that the λ_2 Yukawa coupling is proportional to the electron mass, so that $\lambda_2 = m_e/w_L$. Strictly, the only information that we know about w_L is that $u_1^2 + u_2^2 + w_L^2 \approx (250 \text{ GeV})^2$, so that the most natural value for λ_2 is $\lambda_2 \sim 10^{-5}$. Thus, with this in mind, we have

$$\tau_N \sim \frac{1}{\lambda_1^2 \tilde{\lambda}_1^2} \left(\frac{10^{-5}}{\lambda_2} \right)^4 \left(\frac{\text{TeV}}{w_R} \right)^2 \left(\frac{m_{\chi_R}}{\text{TeV}} \right)^4 \left(\frac{m_{\chi_L}}{\text{TeV}} \right)^8 10^{21} \text{ yr.} \quad (13)$$

The current experimental bound on the (bound) neutron decay mode $N \rightarrow e \bar{e} \nu$ is $\tau_N \gtrsim 7 \times 10^{31} \text{ yr}$ at 90% c.l. [8]. This bound suggests $\lambda_1 \tilde{\lambda}_1 \lesssim 10^{-5}$, which is not a very stringent limit. Thus, clearly this model is not significantly constrained by current limits on nucleon decay. Obviously, if a $N \rightarrow e \bar{e} \nu$ signal were to be experimentally observed, then this would be compatible with this model. Finally, note that we have implicitly assumed that the scalars χ_L, χ_R coupled the first generation quarks, u, d with the first generation leptons ν_e, e . It is possible that this is not the case. If the scalars χ_L, χ_R coupled the first generation quarks u, d with the second generation leptons ν_μ, μ , then the decay $N \rightarrow \nu_\mu \mu \bar{\mu}$ would be the dominant decay mode. Note that the decay rate for this mode might be somewhat larger due to the larger λ_2 . The experimental bound is only slightly weaker, $\tau_N \gtrsim 4 \times 10^{31} \text{ yr}$ at 90% c.l. [8], so the bound on $\lambda_1 \tilde{\lambda}_1$ is somewhat stronger, but certainly cannot exclude a symmetry breaking scale of the order of a TeV.

IV. GAUGE INTERACTION MEDIATED RARE DECAYS

In the alternative 422 model the right-handed leptons belong to the same multiplet as the right-handed quarks. This means that there will be gauge interactions of the form

$$\mathcal{L} = \frac{g_s}{\sqrt{2}} \bar{D}_R^i W_\mu' \gamma^\mu K'^{ij} l_R^j + \text{H.c.}, \quad (14)$$

where the latin index is a family index (so that $D_R^1 = d_R, l_R^1 = e_R, D_R^2 = s_R$, etc.), the W_μ' is the colored electrically charged 2/3 vector gauge boson and K'^{ij} is a Cabibbo-Kobayashi-Maskawa (CKM) type matrix. In Ref. [4] it was shown that an approximately diagonal K' matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (15)$$

would lead to $K^0 \rightarrow \mu^\pm e^\mp$ decay faster than the experimental limit unless $m_{W'} \gtrsim 140$ TeV. However, as was discussed in Ref. [4] there is no compelling reason why K' must be diagonal, and it was shown that if K' had the approximate form

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (16)$$

then the primary constraint on the model is from $B^0 \rightarrow \mu^\pm e^\mp$ rare decays.⁶ In this case the experimental bound on the $SU(4)$ symmetry breaking scale is much weaker, $m_{W'} \gtrsim 1$ TeV [4]. Our purpose now is to examine all possible forms for the matrix K' which can lead to such low symmetry breaking scales. As discussed already in the introduction, a TeV symmetry breaking scale is theoretically suggested to avoid a gauge hierarchy and also to make the model accessible to experiments. Clearly, the rare decays $K_L \rightarrow \mu^\pm e^\mp$ must be suppressed sufficiently for a TeV symmetry breaking scale to occur, and this implies that the only possible (approximate) forms for K' are

$$\begin{aligned} K'_1 &= \begin{pmatrix} 0 & 0 & 1 \\ \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \end{pmatrix}, \\ K'_2 &= \begin{pmatrix} \cos \beta & \sin \beta & 0 \\ 0 & 0 & 1 \\ -\sin \beta & \cos \beta & 0 \end{pmatrix}, \\ K'_3 &= \begin{pmatrix} \cos \gamma & 0 & \sin \gamma \\ -\sin \gamma & 0 & \cos \gamma \\ 0 & 1 & 0 \end{pmatrix}, \\ K'_4 &= \begin{pmatrix} 0 & \cos \delta & \sin \delta \\ 0 & -\sin \delta & \cos \delta \\ 1 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (17)$$

If $K' = K'_i$ ($i=1, \dots, 4$) then the rare decays $K_L \rightarrow \mu^\pm e^\mp$ are avoided because the W' connects either the d quark or s quark with the tau lepton. However, as we will discuss in detail in a moment, in each case rare B^0 decays will occur. The relevant experimental limits are (at 90% c.l.) [8]

$$B(B_0 \rightarrow \tau^\pm e^\mp) < 5.3 \times 10^{-4},$$

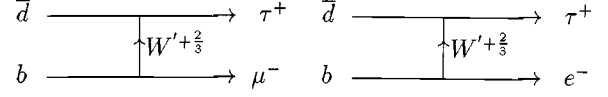
⁶Such nonstandard K' matrices have also been studied in the context of the usual Pati-Salam type model, see Ref. [7].

$$B(B_0 \rightarrow \tau^\pm \mu^\mp) < 8.3 \times 10^{-4},$$

$$B(B_0 \rightarrow \mu^\pm e^\mp) < 5.9 \times 10^{-6}, \quad (18)$$

We now discuss the four possible cases $K' = K'_i$ in turn

(1) If $K' \simeq K'_1$, then $B^0 \rightarrow \tau^+ \mu^-$ and $B^0 \rightarrow \tau^+ e^-$ may occur, which are mediated by the following Feynman diagrams:



The decay rate for $B^0 \rightarrow \tau^+ \mu^-$, assuming maximal μ production for $\alpha=0$, is calculated from the above Feynman diagram. This diagram corresponds (after a Fierz rearrangement) to the following effective 4 fermion Lagrangian density,

$$\mathcal{L}^{\text{eff}} = \frac{G_X}{\sqrt{2}} \bar{d} \gamma_\mu (1 + \gamma_5) b \bar{\mu} \gamma^\mu (1 + \gamma_5) \tau + \text{H.c.}, \quad (19)$$

where $G_X \equiv \sqrt{2} g_s^2(m_{W'})/8m_{W'}^2$. From this effective Lagrangian density it is straightforward to calculate the decay rate,

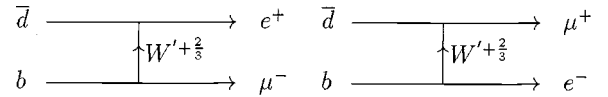
$$\Gamma(B^0 \rightarrow \tau^+ \mu^-) = \frac{G_X^2 f_B^2}{8\pi} m_B m_\tau^2. \quad (20)$$

Evaluating this using $f_B \approx 150$ MeV, $m_B \approx 5.3$ GeV, and using the measured total decay rate, we find the branching fraction,

$$B(B^0 \rightarrow \tau^+ \mu^-) \approx 10^{-3} \left(\frac{\text{TeV}}{m_{W'}} \right)^4. \quad (21)$$

Thus, from the experimental limits, Eq. (18), we see that $m_{W'} \gtrsim 1$ TeV. Similar bounds also occur for other values of α . Note that in the case where $\alpha \approx \pi/2$ the bound comes from the $B^0 \rightarrow \tau^+ e^-$ decay.

(2) If $K' \simeq K'_2$, then $B^0 \rightarrow e^\pm \mu^\mp$ decays can occur via the following Feynman diagrams:



The decay rate for the first process is proportional to $\cos^4 \beta$ and for the second process it is proportional to $\sin^4 \beta$. The Feynman diagrams can easily be evaluated as before, the only difference is that $m_\tau^2 \rightarrow m_\mu^2$ in Eq. (20). Taking case $\beta = 0$,

TABLE I. A summary of results for K' .

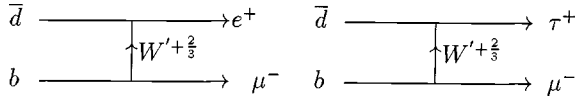
Matrix	Process	Bound
$K'_1; \alpha=0$	$B^0 \rightarrow \mu^- \tau^+$	$W' \gtrsim 1 \text{ TeV}$
$K'_1; \alpha=\pi/2$	$B^0 \rightarrow e^- \tau^+$	$W' \gtrsim 1 \text{ TeV}$
$K'_2; \beta=0$	$B^0 \rightarrow e^+ \mu^-$	$W' \gtrsim 1 \text{ TeV}$
$K'_2; \beta=\pi/2$	$B^0 \rightarrow e^- \mu^+$	$W' \gtrsim 1 \text{ TeV}$
$K'_3; \gamma=0$	$B^0 \rightarrow e^+ \mu^-$	$W' \gtrsim 1 \text{ TeV}$
$K'_3; \gamma=\pi/2$	$B^0 \rightarrow \tau^+ \mu^-$	$W' \gtrsim 1 \text{ TeV}$
$K'_4; \delta=0$	$B^0 \rightarrow \mu^+ e^-$	$W' \gtrsim 1 \text{ TeV}$
$K'_4; \delta=\pi/2$	$B^0 \rightarrow \tau^+ e^-$	$W' \gtrsim 1 \text{ TeV}$
$K'_4; \delta=\pi/4$	$K^0 \rightarrow \mu^+ \mu^-$	$W' \gtrsim 30 \text{ TeV}$

$$\Gamma(B^0 \rightarrow e^+ \mu^-) = \frac{G_X^2 f_B^2}{8\pi} m_B m_\mu^2,$$

$$\Rightarrow B(B^0 \rightarrow e^+ \mu^-) = 3 \times 10^{-6} \left(\frac{\text{TeV}}{m_{W'}} \right)^4. \quad (22)$$

Thus, comparing the above rate with the experimental lower limit in Eq. (18) we see that the limit on the W' mass is also about 1 TeV in this case (similar bounds also occur for other values of β).

(3) If $K' \approx K'_3$, then $B^0 \rightarrow \mu^- e^+$ and $B^0 \rightarrow \mu^- \tau^+$ decays can occur via the following Feynman diagrams:



The rate for the first process is proportional to $\cos^2 \gamma$ and for the second process it is proportional to $\sin^2 \gamma$. These processes are similar to cases already discussed, and the lower bound in this case is therefore also about 1 TeV.

(4) If the $K' \approx K'_4$ then $B^0 \rightarrow \mu^+ e^-$ and $B^0 \rightarrow \tau^+ e^-$ decays can occur, and the bound from these decays, being similar to processes already discussed is also about a TeV. However, in this case there is another possible rare decay which is $K_L \rightarrow \mu^+ \mu^-$. This decay rate is proportional to the factor $\sin^2 \delta \cos^2 \delta$,

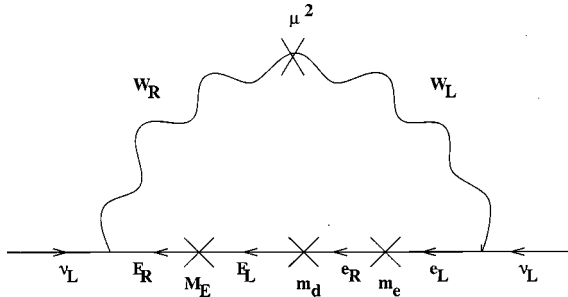


FIG. 2. One-loop Feynman diagram which leads to small neutrino Majorana mass term. [The $W_L W_R$ mixing mass squared is obtained from $\mathcal{L} = (D_\mu \langle \phi \rangle)^\dagger D^\mu \langle \phi \rangle$ and is given by $\mu^2 = g_R g_L u_1 u_2$.]

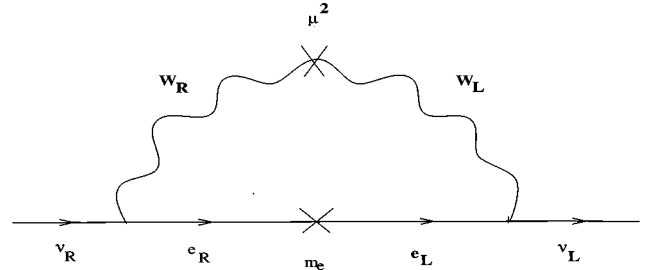


FIG. 3. One-loop Feynman diagram leading to the mass term $\bar{\nu}_L \nu_R$.

$$\Gamma(K_L \rightarrow \mu^\pm \mu^\mp) = \sin^2 \delta \cos^2 \delta \frac{G_X^2 f_K^2}{4\pi} m_K m_\mu^2,$$

$$\Rightarrow B(K_L \rightarrow \mu^\pm \mu^\mp) \approx 5 \times 10^{-3} \left(\frac{\text{TeV}}{m_{W'}} \right)^4$$

$$\text{for } \delta = \frac{\pi}{4}. \quad (23)$$

The measured branching ratio is [8]

$$B(K_L \rightarrow \mu^+ \mu^-) = (7.2 \pm 0.5) \times 10^{-9}. \quad (24)$$

Conservatively, demanding that the W' contribution equation (23) be less than the total branching fraction implies the limit $m_{W'} \gtrsim 30 \text{ TeV}$ for the maximal case where $\delta = \pi/4$. We briefly summarize the main results in Table I.

V. NATURALLY SMALL NEUTRINO MASSES

In the 422 model there are four electrically neutral Weyl states per generation, $\nu_L, \nu_R, E_{L,R}^0$. Thus the masses for the neutral leptons will be described by a 12×12 mass matrix. The $E_{L,R}^0$ gain masses from the large VEV w_R and are expected to be quite heavy (experimentally we know that they must be heavier than about $m_Z/2 \approx 45 \text{ GeV}$). The approximately sterile [i.e., $SU(2)_L \otimes U(1)_Y$ singlet] ν_R states gain masses by mixing with the E leptons (see below for more details). At tree level the ordinary neutrinos (i.e., the ν_L states) are massless. This is quite easy to see, because the masses of the fermions arise from the Lagrangian density,

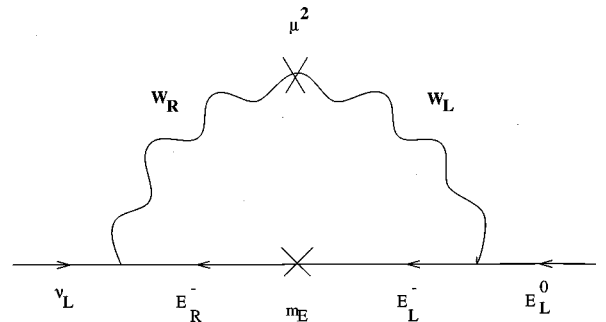


FIG. 4. One-loop Feynman diagram leading to the neutrino mixing term $\nu_L (E_L^0)^c$.

Eq. (5), and the ν_L states do not couple to any VEV.

In order to gain insight into the neutrino masses, let us first consider the toy case of just one generation, with just the usual first generation states (together with the exotic E leptons). In this case the tree level neutrino mass matrix, which can be obtained from Eq. (5), has the form

$$\mathcal{L}_{\text{tree}} = \bar{\psi}_L M (\psi_L)^c + \text{H.c.}, \quad (25)$$

where

$$\psi_L^T = [\nu_L (\nu_R)^c, E_L^0 (E_R^0)^c] \quad (26)$$

and

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & m_u & m_e \\ 0 & m_u & 0 & m_E \\ 0 & m_e & m_E & 0 \end{pmatrix}. \quad (27)$$

Thus, at the tree level the ordinary neutrino ν_L is massless. The “light” sterile ν_R state has mass

$$m_{\nu_R} \simeq \frac{2m_u m_e}{m_E}. \quad (28)$$

At one loop, there are important corrections to this mass matrix. In Ref. [4] only one such correction (m_M) was considered. Here we do a better job by including all possible one-loop gauge corrections involving ν_L . In particular, the mass terms

$$\mathcal{L}_{1\text{-loop}}^{\text{eff}} = m_M \bar{\nu}_L (\nu_L)^c + m_D \bar{\nu}_L \nu_R + m_{\nu E} \bar{\nu}_L (E_L^0)^c + \text{H.c.} \quad (29)$$

are generated from the Feynman diagrams, Figs. 2–4. Evaluating these diagrams,

$$\begin{aligned} m_M &= m_e m_d m_E \frac{g_R g_L}{8\pi^2} \left[\frac{\mu^2}{m_{W_R}^2} \right] \left[\frac{\log(m_{W_R}^2/m_E^2)}{m_{W_R}^2 - m_E^2} - \frac{\log(m_{W_L}^2/m_E^2)}{m_{W_L}^2 - m_E^2} \right], \\ m_D &= m_e \frac{g_R g_L}{8\pi^2} \left[\frac{\mu^2}{m_{W_R}^2} \right] \log \left(\frac{m_{W_R}^2}{m_{W_L}^2} \right), \\ m_{\nu E} &= m_E \frac{g_R g_L}{8\pi^2} \left[\frac{\mu^2}{m_{W_R}^2} \right] \left[\log \left(\frac{m_{W_R}^2}{m_{W_L}^2} \right) + \frac{m_E^2 \log(m_{W_L}^2/m_E^2)}{m_E^2 - m_{W_L}^2} - \frac{m_E^2 \log(m_{W_R}^2/m_E^2)}{m_E^2 - m_{W_R}^2} \right], \end{aligned} \quad (30)$$

where $\mu^2 \equiv g_R g_L u_1 u_2$ is the $W_L - W_R$ mixing mass. Including these radiatively generated mass terms, the effective mass matrix becomes

$$M = \begin{pmatrix} m_M & m_D & m_{\nu E} & 0 \\ m_D & 0 & m_u & m_e \\ m_{\nu E} & m_u & 0 & m_E \\ 0 & m_e & m_E & 0 \end{pmatrix}. \quad (31)$$

The effect of this is to give the neutrino ν_L a small Majorana mass, given approximately by

$$m_\nu \simeq \frac{\text{Det}(M)}{2m_e m_u m_E}, \quad (32)$$

that is,

$$m_\nu \simeq m_M + \frac{m_D^2 m_E}{2m_e m_u} + \frac{m_{\nu E}^2 m_e}{2m_u m_E} - \frac{m_{\nu E} m_D}{m_u}. \quad (33)$$

Actually no precise predictions can be made for the neutrino masses, due, for example, to the unknown masses of the

heavy E_i^0 leptons. Also, we have not included one-loop corrections coming from the Higgs scalars in the model. These corrections may be important, but they are harder to pin down because they depend on unknown Yukawa couplings (c.f., with the gauge corrections which depend on the known gauge couplings). Nevertheless, if we consider the parameter range where the scalar boson corrections can be approximately neglected, then it is possible to show that the neutrino masses are naturally light. From Eq. (5) the VEVs u_1, u_2 can be related to the bottom and top quark masses as follows:

$$m_b = \lambda_3 u_1 + \lambda_4 u_2, \quad m_t = \lambda_3 u_2 + \lambda_4 u_1. \quad (34)$$

It follows that

$$\frac{u_1 u_2}{u_1^2 + u_2^2} \sim \frac{m_b}{m_t}. \quad (35)$$

Hence

$$\frac{\mu^2}{m_{W_R}^2} \sim \frac{1}{2\sqrt{3}} \frac{m_{W_L}^2}{m_{W_R}^2} \frac{m_b}{m_t}, \quad (36)$$

where we have used $g_R \approx g_L / \sqrt{3}$ [4] and $m_{W_L}^2 = \frac{1}{2} g_L^2 (u_1^2 + u_2^2 + w_L^2) \sim \frac{1}{2} g_L^2 (u_1^2 + u_2^2)$. Thus, we have

$$\begin{aligned} m_M &\sim \frac{m_e m_d}{m_E} \frac{g_L^2}{(4\pi)^2} \frac{m_{W_L}^2}{m_{W_R}^2} \frac{m_b}{m_t}, \\ m_D &\sim \frac{m_E}{m_d} m_M, \\ m_{\nu E} &\sim \frac{m_E}{m_e} m_D. \end{aligned} \quad (37)$$

Hitherto we have studied only the one generation case. Of course the full neutral lepton mass matrix will be a 12×12 generalization of Eq. (31). While the general mass matrix is obviously quite complicated, with many free parameters, it is still possible to place an upper limit on the largest possible (ordinary) neutrino mass. This will occur when $m_e \rightarrow m_\tau$ and $m_d \rightarrow m_b$ (with $m_u \rightarrow m_u$, unchanged). In this case,

$$\begin{aligned} m_M|_{\max} &\sim \frac{m_\tau m_b}{m_E} \frac{g_L^2}{(4\pi)^2} \frac{m_{W_L}^2}{m_{W_R}^2} \frac{m_b}{m_t} \\ &\sim 50 \left(\frac{\text{TeV}}{m_{W_R}} \right)^2 \left(\frac{100 \text{ GeV}}{m_E} \right) \text{ eV} \end{aligned} \quad (38)$$

and

$$\begin{aligned} \frac{m_D^2 m_E}{m_\tau m_u} \Big|_{\max} &\sim \frac{m_{\nu E}^2 m_\tau}{m_E m_u} \Big|_{\max} \sim \frac{m_{\nu E} m_D}{m_u} \Big|_{\max}, \\ &\sim m_\tau \left(\frac{g_L^4}{(4\pi)^4} \right) \left(\frac{m_{W_L}^4}{m_{W_R}^4} \right) \frac{m_b^2}{m_t^2} \frac{m_E}{m_u} \\ &\sim 20 \left(\frac{\text{TeV}}{m_{W_R}} \right)^4 \left(\frac{m_E}{100 \text{ GeV}} \right) \text{ eV}. \end{aligned} \quad (39)$$

Thus the upper limit on the neutrino mass is naturally light (i.e., less than about 50 eV) despite the low TeV symmetry breaking scale of the model. (Of course it may be possible to evade this if scalar boson corrections are considered.) Naturally all three neutrinos may be considerably lighter than this maximum mass; such information will depend on the parameters of the full 12×12 neutral lepton mass matrix. Observe also that in addition to three light neutrinos, the model has three heavier sterile neutrinos: the ν_R 's, and the heavy leptons E_i^0 .

VI. CONCLUSION

We have studied the alternative $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ gauge model which allows unification of the quarks and leptons at the TeV scale. We have shown that the leading nucleon decay mode in this model is (bound) neutron decay, $N \rightarrow \nu l \bar{l}$ (where $l = e, \mu$). While current experimental bounds on bound neutron decay cannot exclude a TeV symmetry breaking scale, such experimental searches can potentially test the model. More important tests are expected to come from the up-coming B factory experiments. From improved limits (or discoveries) of rare B decays, such as $B^0 \rightarrow e^\pm \mu^\mp$, $B^0 \rightarrow e^\pm \tau^\mp$, and $B^0 \rightarrow \mu^\pm \tau^\mp$, much of the most interesting region of parameter space where the $SU(4)$ symmetry breaking scale is in the TeV range will be covered. Finally, the neutrino masses are radiatively generated and are naturally quite light.

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